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Russian Translation

One-Dimensional Self-Similar Motions of a Conducting Gas in a Magnetic Field

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1. Let us consider one-dimensional, nonstationary adiabatic motions of a perfect electroconducting gas with cylindrical and plane waves. The magnetic field $\vec{H}$ is directed perpendicularly to the particle motion trajectories (along the axis of symmetry in the cylindrical case or along the tangent to concentric circles with a center on this axis). The conductivity of the gas is considered to be infinite; we neglect the viscosity and heat conduction. Under the assumptions made, the motion equations are

$$\begin{aligned}
&\frac{dv}{dt} = -\frac{1}{\rho}\left[\frac{\partial}{\partial r}(p+h) + \frac{2h}{r}(1-n)\right]\\
&\frac{d\rho}{dt} = -\rho\left[\frac{\partial v}{\partial r} + (\nu - 1)\frac{v}{r}\right]\\
(1)\qquad &\frac{dp}{dt} = -\gamma p\left[\frac{\partial v}{\partial r} + (\nu - 1)\frac{v}{r}\right]\\
&\frac{dh}{dt} = -2h\left[\frac{\partial v}{\partial r} + (\nu - 1)\frac{nv}{r}\right]
\end{aligned}$$

where $\frac{dv}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}$ etc., $h = \frac{H^2}{8\pi}$; H is the magnetic field intensity; $\nu = 2,1$; $n = 0$ for the cylindrical magnetic field, $n = 1$ for a field directed along the axis of symmetry ($\nu = 2$); the rest of the notation is as customary. Since equations (1) do not contain any dimensional constants, the motion will be self-similar [1] if only two dimensional constants with independent dimensionalities enter into the initial and boundary conditions of the problem.

L. I. Sedov [1] developed the theory of unsteady, self-similar motion of a fluid and gas in the absence of a magnetic field. Let us use the L. I. Sedov notation and terminology.

Let there be two constants a and b with independent dimensionalities among the defining parameters, where $[a] = ML^kT^s$, $[b] = LT^{-\delta}$.

Let us introduce the nondimensional variables

$$v = \frac{r}{t}V;\quad \rho = \frac{a}{r^{k+3}t^s}R;\quad p = \frac{a}{r^{k+1}t^{s+2}}\mathcal{P};\quad h = \frac{a}{r^{k+1}t^{s+2}}\mathcal{H};\quad \lambda = \frac{r}{bt^\delta}$$

The nondimensional functions $V, R, \mathcal{P}, \mathcal{H}$ depend only on the one nondimensional variable λ because of self-similarity. The partial differential equation system (1) is equivalent to the following system of ordinary differential equations:

$$(2)\qquad \lambda\left[(V-\delta)V+\frac{(\mathcal{H}+\mathcal{P})'}{R}\right]=-V^2+V+(k+1)\frac{\mathcal{H}+\mathcal{P}}{R}-\frac{2(1-n)\mathcal{H}}{R}$$

$$(3)\qquad \lambda\left[(V-\delta)\frac{R'}{R}+V'\right]=s+(k-\nu+3)V$$

$$(4)\qquad \lambda\left[(V-\delta)\frac{\mathcal{P}'}{\mathcal{P}}+\gamma V'\right]=s+2+(k+1-\gamma\nu)V$$

$$(5)\qquad \lambda\left[(V-\delta)\frac{\mathcal{H}'}{2\mathcal{H}}+V'\right]=\frac{s+2}{2}+\left[\frac{k-1}{2}+(1-\nu)n\right]V$$

If the constants , δ are related to s, k, ν, n by means of the relations

$$2(\gamma\nu-k)=(2-\nu+\gamma\nu)(\delta+s+2)\ ;\qquad 2-\gamma\nu-2(1-\nu)n=0$$

then the system (2) - (5) has the particular solution

$$V=A\ ;\qquad \mathcal{P}=B\lambda\ ;\qquad \rho=C\lambda\ ;\qquad \mathcal{H}=D\lambda$$

where B and D are arbitrary positive constants;

$$A=\frac{2}{2-\nu+\gamma\nu}\ ;\qquad C=\frac{k(B+D)-2(1-n)D}{A(A-1)}$$

The dimensional velocity for this solution depends linearly on the coordinate r. More general solutions of this type are analyzed in [2,3].

Using the conservation laws and the methods of dimensional analysis [1,4], it can be shown that equations (3) - (5) have two algebraic integrals:

1) Adiabatic integral [1,5]

$$(6)\qquad \frac{\mathcal{P}}{R^{\gamma}}=[R(V-\delta)]^{\frac{2-(\gamma-1)s+\delta[k+1-\gamma(k+3)]}{\mu}}\ \lambda^{-\frac{[2+\nu(\gamma-1)]s+2(k+3-\nu)}{\mu}}\ \varkappa_1$$

2) Frozen integral

$$(7)\qquad \frac{\sqrt{\mathcal{H}}}{\lambda^{1-n}R}=[R(V-\delta)]^{\frac{2-s-\delta(k+2n+3)}{2\mu}}\ \lambda^{-\frac{(k+5)\mu-(\nu-k-3)[2-s-\delta(k+2n+3)]}{2\mu}}\ \varkappa_2$$

where $\varkappa_1$ and $\varkappa_2$ are arbitrary constants; $\mu=s+\delta(k+3-\nu)$.

Hence, the solution of all the self-similar problems reduces to the integration of a system of two ordinary equations.

If $s+2-\delta(\nu-1-k)=0$, the following energy integral exists for the system (2) - (5)

$$(8)\qquad \lambda^{\nu+2}\left[(\mathcal{P}+\mathcal{H})V-(V-\delta)\left(\frac{RV^2}{2}+\frac{\mathcal{P}}{\gamma-1}+\mathcal{H}\right)\right]=\text{const}$$

The problem reduces to the solution of one equation in this case.

Shockwaves can arise in the gas motion. The conditions on the shocks, which are consequences of the conservation laws, can be written thus for the self-similar motions under consideration:

$$
(9) \qquad
\begin{gathered}
\{R(V-\delta)\} = 0 \\
\{\mathcal{H}(V-\delta)^2\} = 0 \\
\{R(V-\delta)V + \mathcal{P} + \mathcal{H}\} = 0 \\
\left\{R(V-\delta)\left(\frac{\mathcal{P}}{R(\gamma-1)} + \frac{V^2}{2} + \frac{\mathcal{H}}{R}\right) + (\mathcal{P}+\mathcal{H})V\right\} = 0
\end{gathered}
$$

Here, it has been taken into account that there is the following dependence $c = \frac{\delta r_2}{t}$ for the shockwave velocity c, where r_2 is the shockwave radius. The braces denote the difference in the values of the quantities on both sides of the surface of discontinuity. For a flow with shockwaves, (9) are the boundary conditions to find the functions $V(\lambda)$, $R(\lambda)$, $\mathcal{P}(\lambda)$, $\mathcal{H}(\lambda)$.

2. The following self-similar problems can be mentioned, whose solution reduces to the integration of the system (2) - (5).

1) The problem of the motion of a conducting gas according to given initial data (Cauchy problem). It follows from the self-similarity requirement that the initial distributions (for $t = 0$) will have the form:

$$
v_o = \alpha_1 b^{\frac{1}{\delta}} r^{1-\frac{1}{\delta}} ; \qquad \rho_o = \alpha_2 a b^{\frac{s}{\delta}} r^{-(k+3+\frac{s}{\delta})}
$$

$$
p_o = \alpha_3 a b^{\frac{s+2}{\delta}} r^{-(k+1+\frac{s+2}{\delta})} ; \qquad h_o = \alpha_4 p_o
$$

where α_i $(i = 1,\ldots,4)$ are assigned dimensional constants. The initial data can undergo a discontinuity at $r = 0$ in the plane case.

The simplest example of this problem is the problem of the disintegration of an arbitrary discontinuity when the following constants are given for $t = 0$:

$$
\begin{aligned}
&v = v_1 ; \quad p = p_1 ; \quad \rho = \rho_1 ; \quad h = h_1 \quad \text{for} \quad r < 0 \\
&v = v_2 ; \quad p = p_2 ; \quad \rho = \rho_2 ; \quad h = h_2 \quad \text{for} \quad r > 0
\end{aligned}
$$

the velocities v_1 and v_2 are directed oppositely, their differences are large in absolute value. Shockwaves, which move with a constant velocity and have a constant intensity, are propagated on both sides as a result of the disintegration of the arbitrary discontinuity.

Using the conditions (9), all the characteristics of the motion which arises can be calculated. A motion of such a kind can occur, for example, in the collision of cosmic gaseous masses. A detailed solution of this problem for $h = 0$ has been given in [7].

2) The problem of the motion of a plane or cylindrical (conducting) piston in a gas. At the initial instant, $v_1 = 0$, p_1, ρ_1 and h_1 are constants and the piston starts to move with the constant velocity U . The problem is self-similar.

Let us consider the solution for the plane piston. A shockwave with the constant velocity c is propagated over the gas in front of the piston. In the region of the motion beyond the shockwave front, $v = v_2 = U$; $p = p_2$, $\rho = \rho_2$, $h = h_2$ are constants.

Using the conditions on the shockwave (9), the dependence of c, p, ρ, h on the piston velocity U and ρ_1, p_1, h_1 can be found. Shown on figure 1 is the dependence of $\frac{c}{a_{*1}}$ on $\frac{U}{a_{*1}}$ for $\gamma = 1.4$ and $\frac{h_1}{p_1} = 0$ and $\frac{h_1}{p_1} = 1$, where $a_{*1}^2 = \frac{\gamma p_1}{\rho_1} + \frac{2h_1}{\rho_1}$.

3) The problem of a strong explosion (electric discharge). At time $t = 0$, there occurs in the gas at rest an instantaneous liberation of the finite energy E_0 along a line, i.e., an explosion occurs; E_0 is calculated per unit length. This explosion can be considered as a high intensity electrical discharge in a gas, originating along a line. The initial density and the magnetic field intensity are variable:

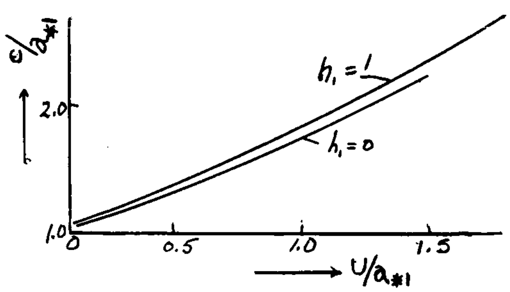

Figure 1

$$\rho_1 = A_1 r^{-\omega} ; \quad h_1 = B_1 r^{-2} \qquad \omega < 3$$

The influence of the initial pressure p_1 can be neglected for a strong explosion [1]. Let us note that the mentioned value h_1 for any constant p_1 satisfies the equilibrium equation $\frac{\partial}{\partial r}(p + h) + \frac{2h}{r} = 0$ for a cylindrical field.

The energy integral (8) exists in this problem, i.e., its solution reduces to the integration of one ordinary first order differential equation.

By analogy with [1], formulations of self-similar problems of detonation and combustion in a gas in the presence of a magnetic field are also possible. For $\nu = 2$, formulations can be given of self-similar problems with magnetic lines of force having the shape of screw lines.

V. A. Steklov Math. Inst. AN USSR March 28, 1958

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